Machine Learning Classification over Encrypted Data

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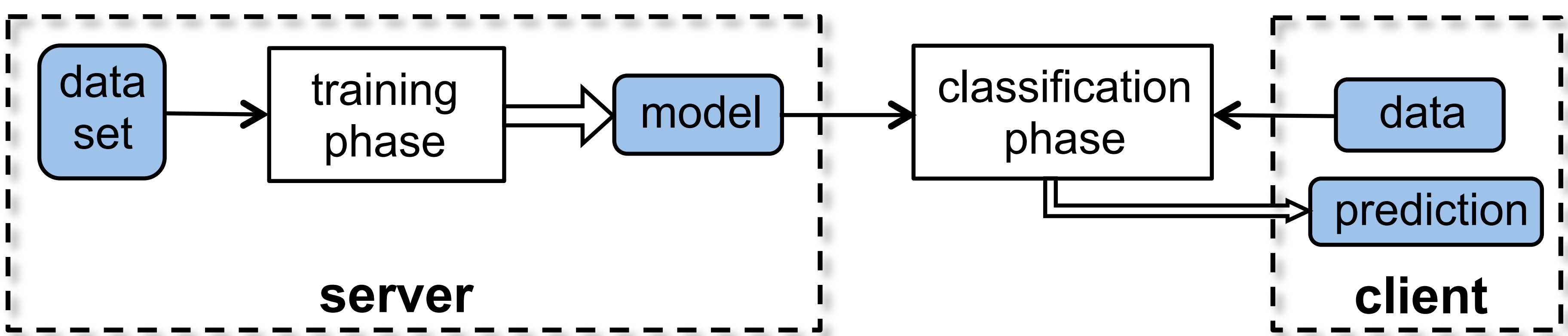
Classification
(Machine Learning)

- Supervised learning (training)
- Classification



Secure Classification

- The provider's model is sensible

  financial model, genetic sequences, …

- Client's private data

  medical records, credit history, …

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  financial model, genetic sequences, …

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MPC / 2PC

Using General 2PC ?

+ Works for every circuit

+ Constant number of interactions

- Have to build circuits

- Hard to ‘compose’

- Not easily reusable

➡ *Ad Hoc* protocols

Scope of our work

- Secure classification, no learning
  the model is already known
- Differential privacy is out of scope
  can be treated separately
- Classifiers as specialized 2PC, but not a specialized classifier

Approach

- Security model: passive (honest-but-curious) adversary
- Identify and construct reusable building blocks
- Practical performance as a primary goal
- Choose the best fitted primitives

  Homomorphic Encryption, FHE, Garbled Circuits, …

Building Blocks

- Dot product
- Encrypted Comparison
- Encrypted (arg)max
- Decision trees
- Encryption scheme switching

Argmax

- Alice $(\llbracket a_1 \rrbracket, \ldots, \llbracket a_n \rrbracket, \mathsf{PK})$
- Bob SK
- The comparison pattern must not depend on the values

Argmax

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- The comparison pattern must not depend on the values
    - Compare everything

Argmax

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  - Compare everything $\Rightarrow O(n^2)$

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- The comparison pattern must not depend on the values
  - ~~Compare everything~~ $\Rightarrow O(n^2)$
  - 'Classical' algorithm

Argmax

- Alice $([\![a_1]\!], \ldots, [\![a_n]\!], \mathsf{PK})$

- Bob SK

- The comparison pattern must not depend on the values

  - ~~Compare everything~~ $\Rightarrow O(n^2)$

  - 'Classical' algorithm $\Rightarrow O(n)$

Compare & Swap

Alice

$(\mathsf{PK}, [\![v]\!], [\![w]\!])$

$[\![\max(v, w)]\!]$

Bob SK

$(v < w)$

Compare & Swap

Alice

$(\mathsf{PK}, [\![v]\!], [\![w]\!])$

$\varnothing$

$[\![\max(v, w)]\!]$

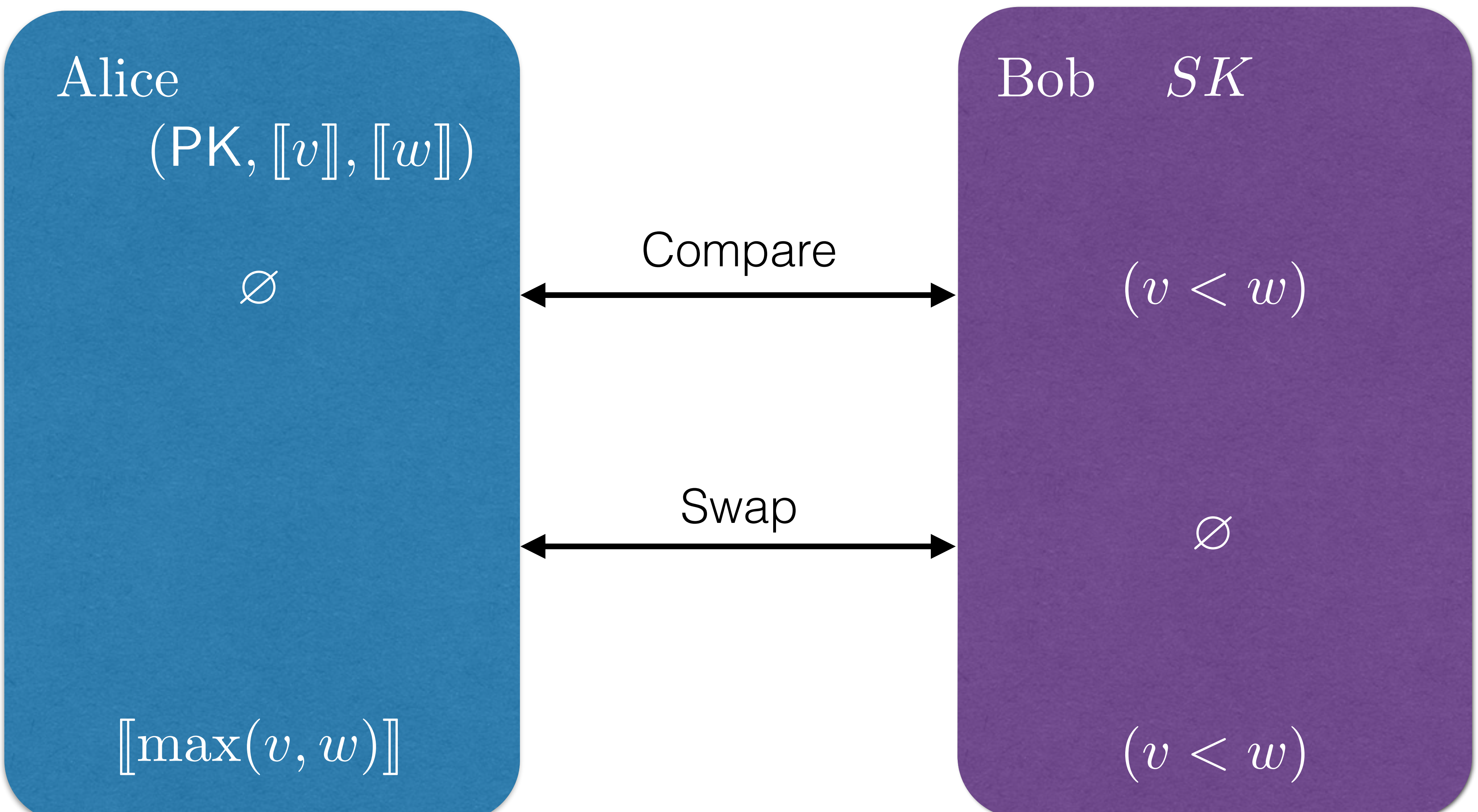
$\xleftrightarrow{\text{Compare}}$

Bob SK

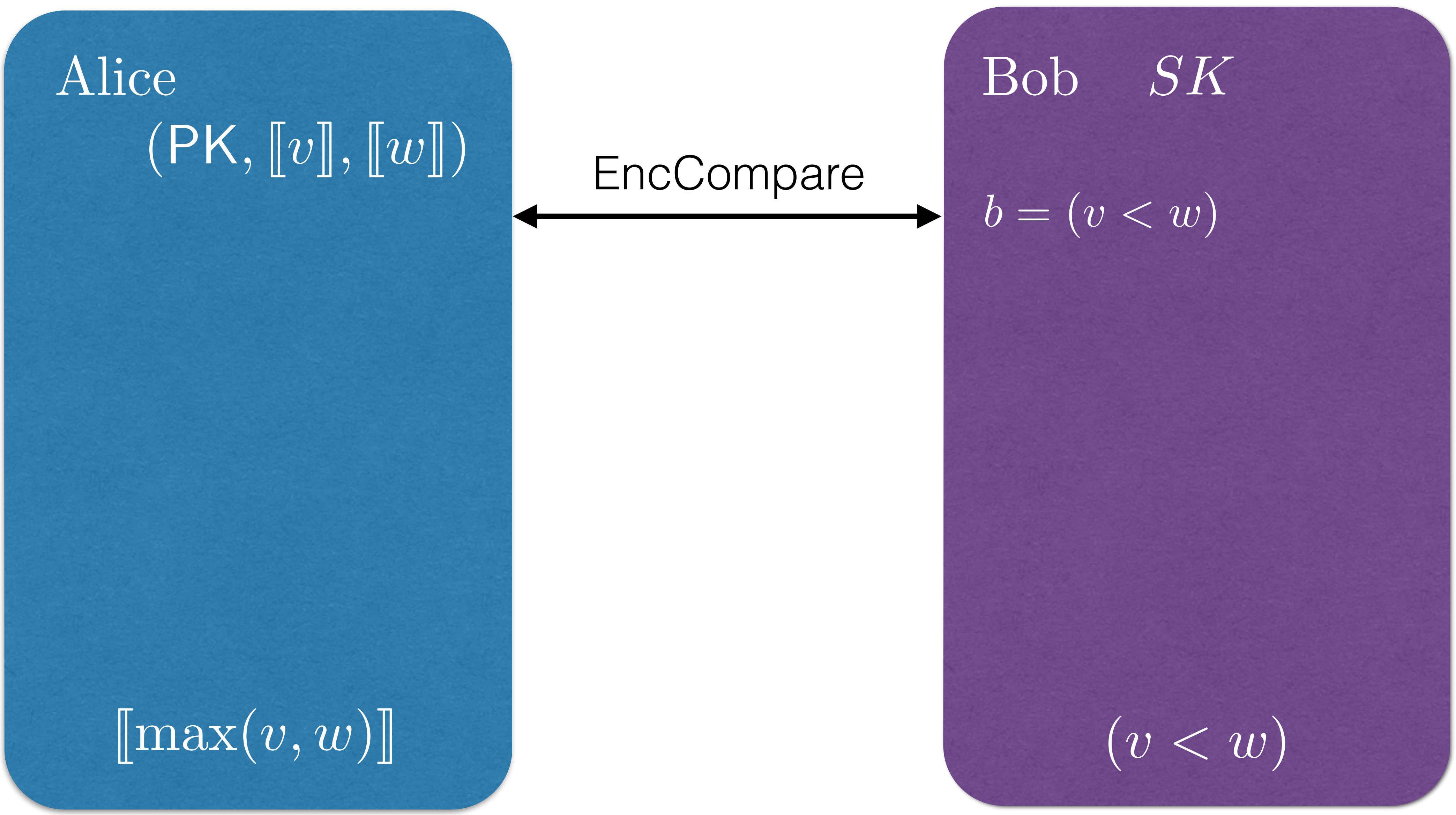
$(v < w)$

$(v < w)$

Compare & Swap



Compare & Swap



Compare & Swap

Alice

$(\mathsf{PK}, [\![v]\!], [\![w]\!])$

$(r, s) \leftarrow M^2$

$[\![v']\!] = [\![v + r]\!]$

$[\![w']\!] = [\![w + s]\!]$

$[\![\max(v, w)]\!]$

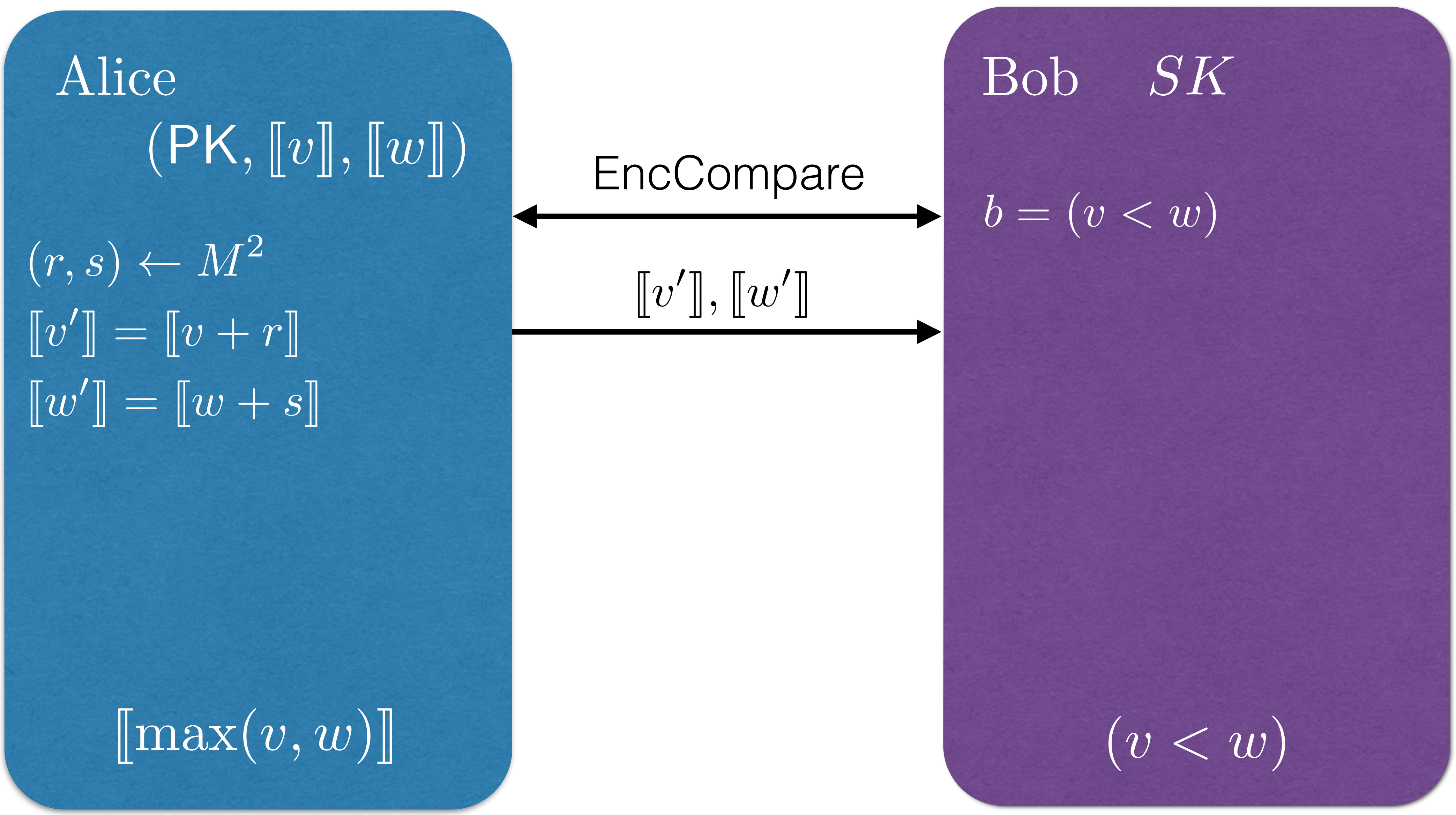
EncCompare

Bob SK

$b = (v < w)$

$(v < w)$

Compare & Swap



Compare & Swap

Alice

$(\mathsf{PK}, \llbracket v \rrbracket, \llbracket w \rrbracket)$

$(r, s) \leftarrow M^2$

$\llbracket v' \rrbracket = \llbracket v + r \rrbracket$

$\llbracket w' \rrbracket = \llbracket w + s \rrbracket$

$\llbracket \max(v, w) \rrbracket$

$\xleftrightarrow{\text{EncCompare}}$

$\xrightarrow{\llbracket v' \rrbracket, \llbracket w' \rrbracket}$

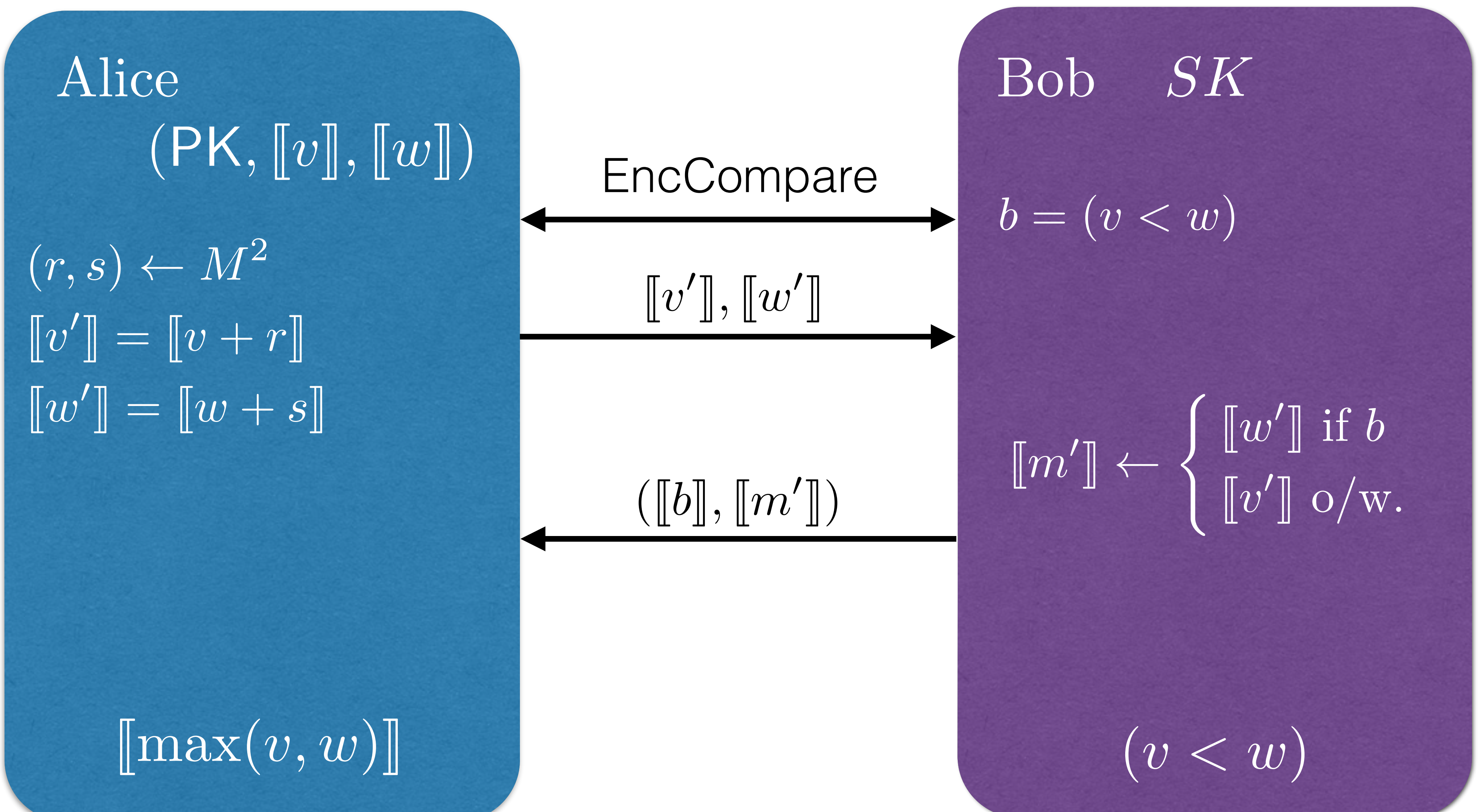
Bob SK

$b = (v < w)$

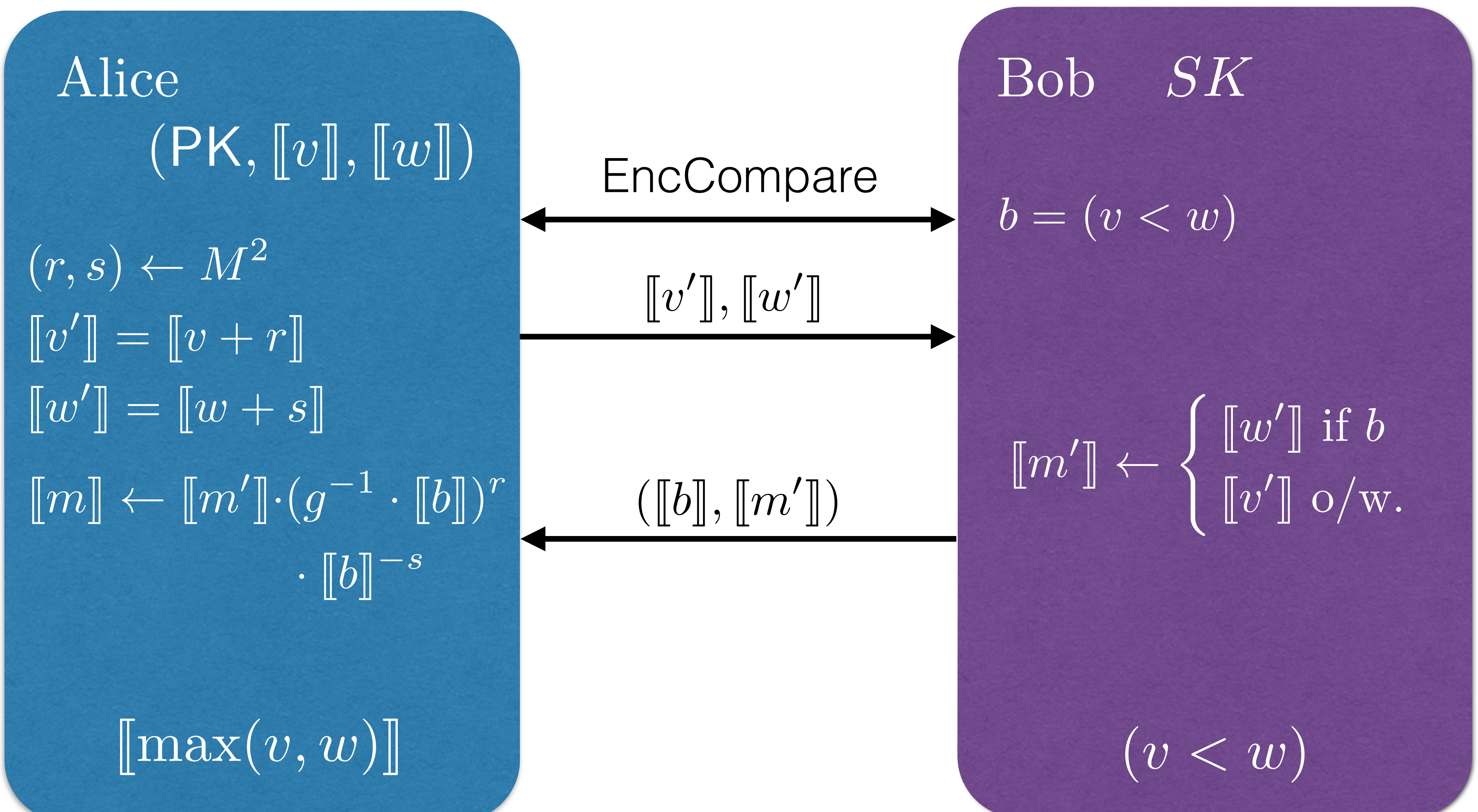
$$\llbracket m' \rrbracket \leftarrow \begin{cases} \llbracket w' \rrbracket & \text{if } b \\ \llbracket v' \rrbracket & \text{o/w.} \end{cases}$$

$(v < w)$

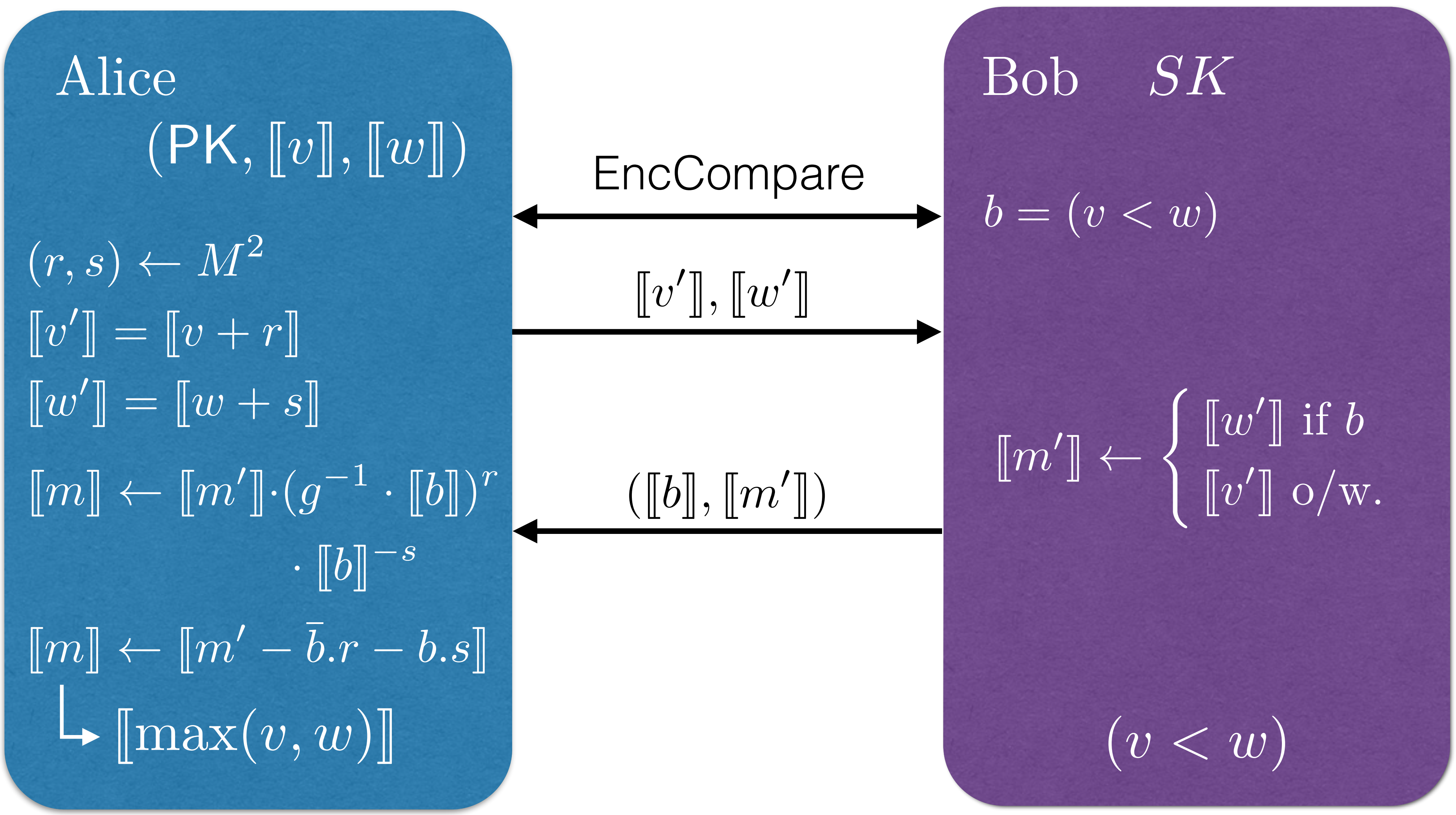
Compare & Swap



Compare & Swap



Compare & Swap



Argmax

- Protocol : n-1 Compare & Swap

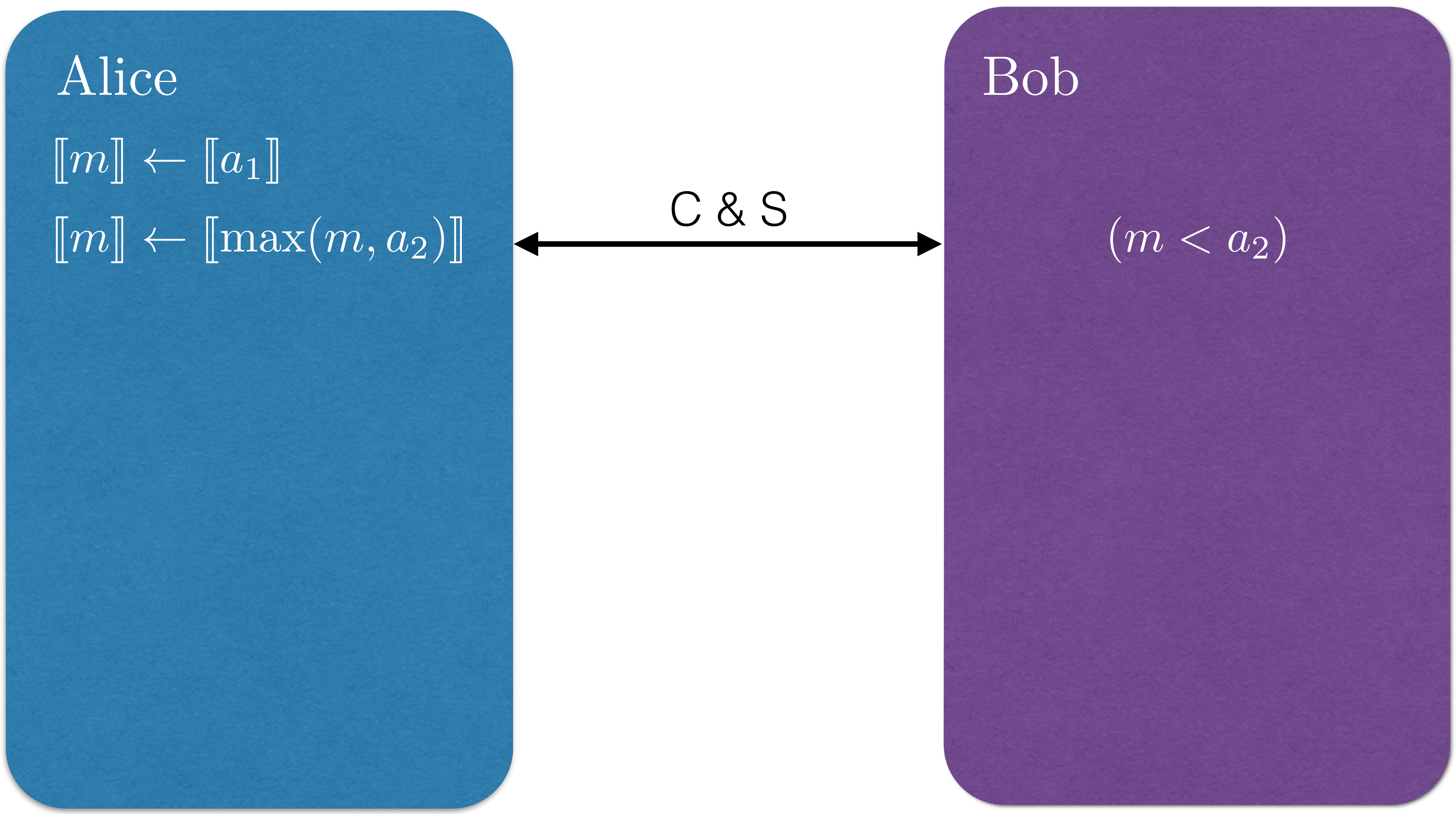
Alice

$[\![m]\!] \leftarrow [\![a_1]\!]$

Bob

Argmax

- Protocol : n-1 Compare & Swap



Argmax

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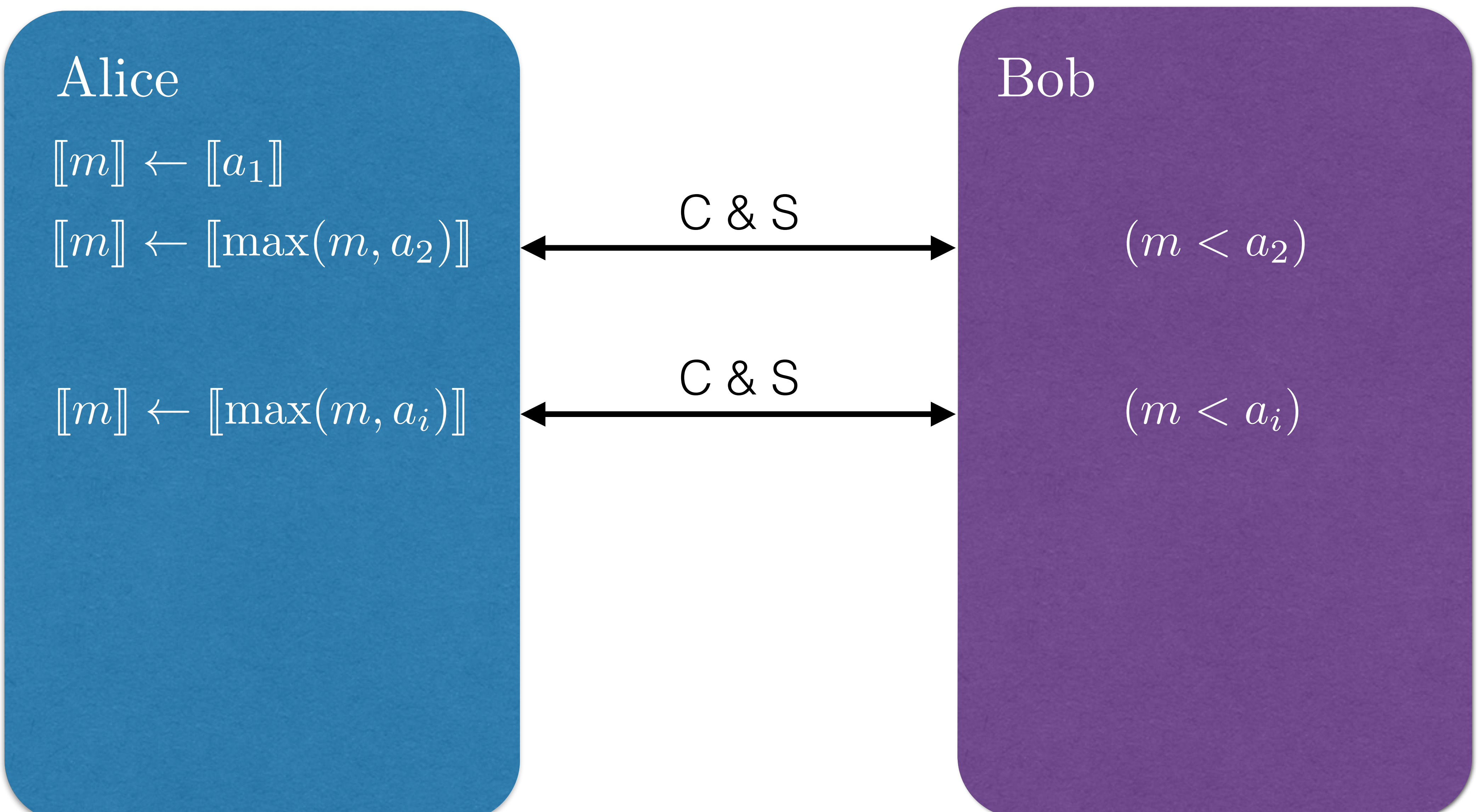
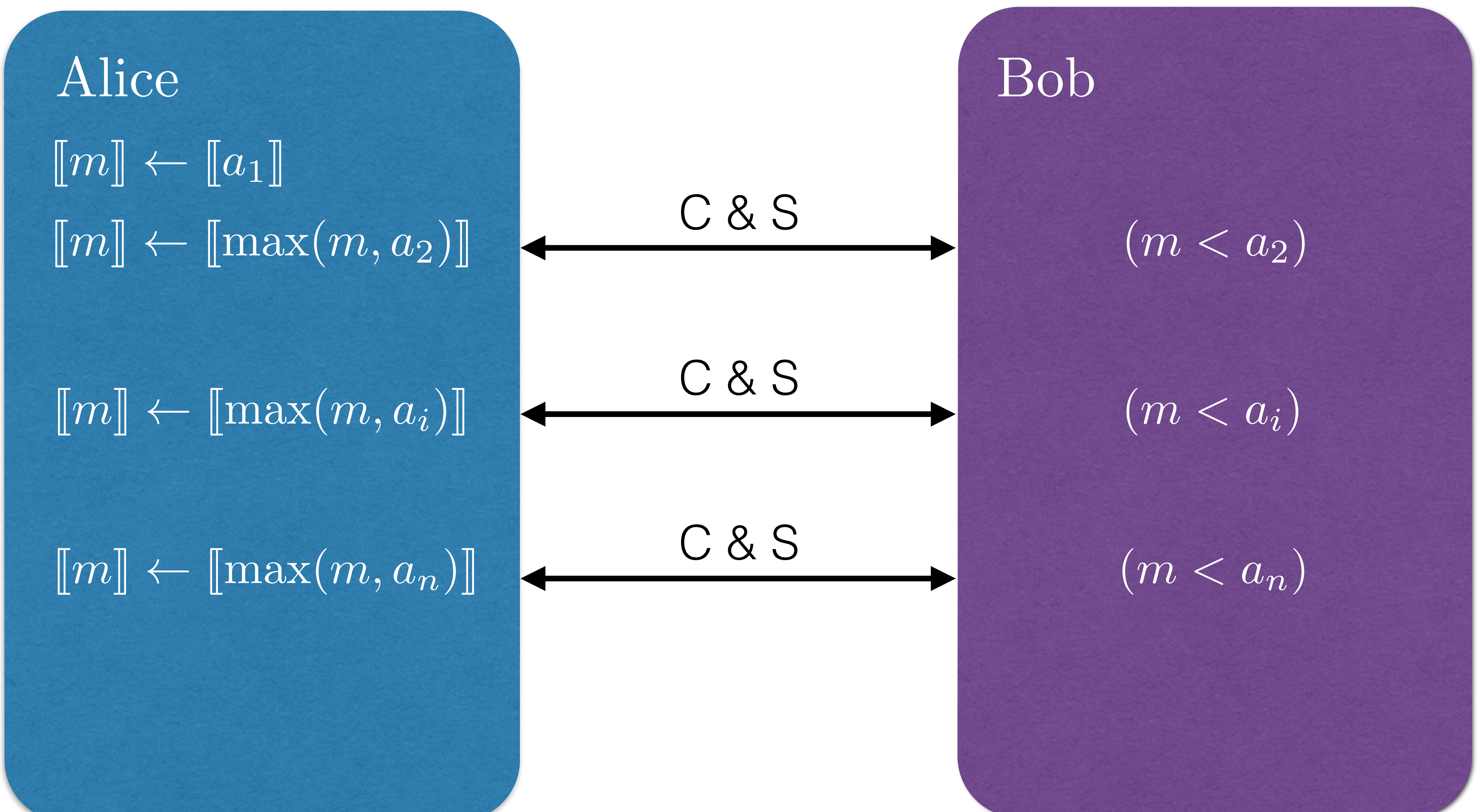


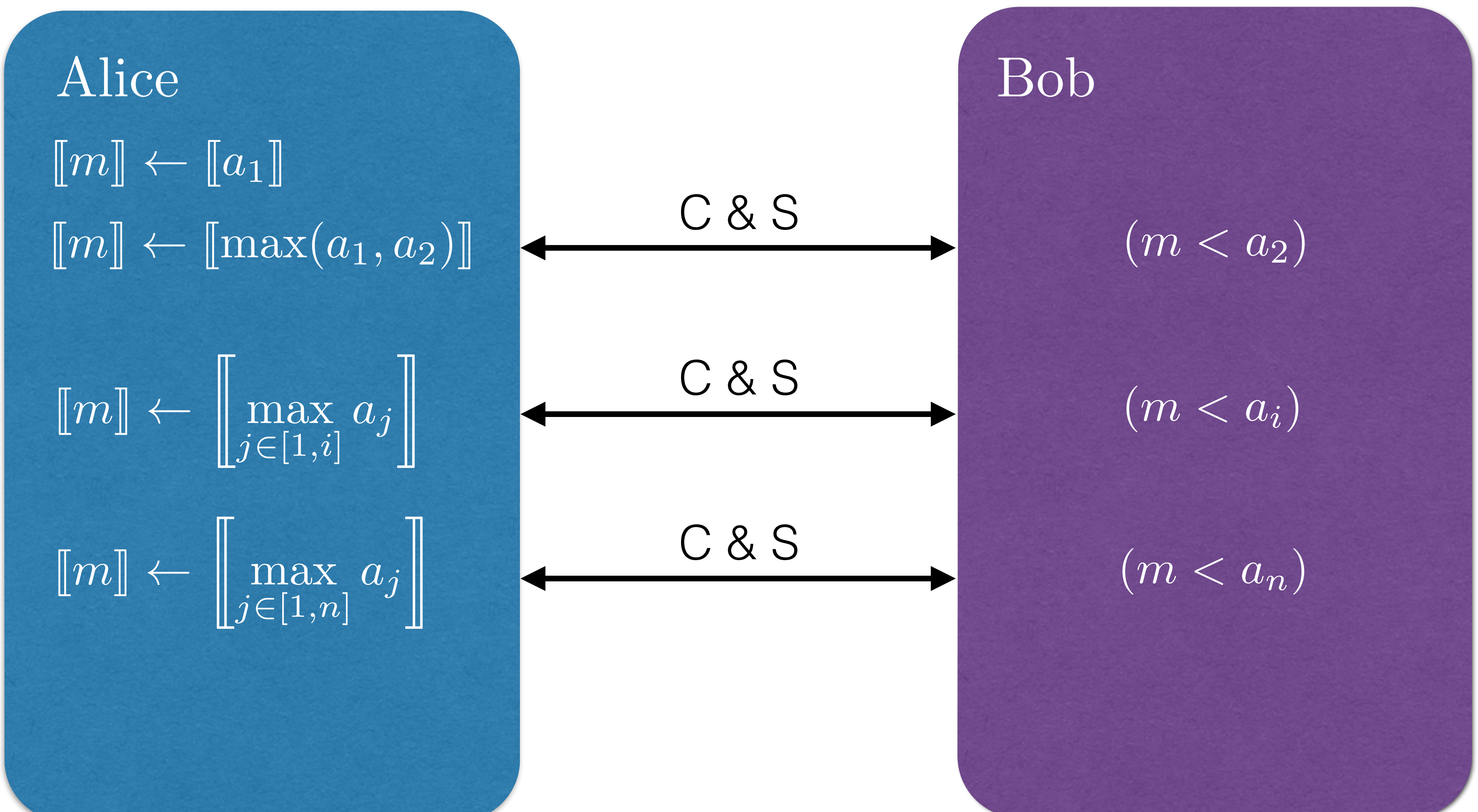
Argmax

- Protocol : n-1 Compare & Swap



Argmax

- Protocol : n-1 Compare & Swap



Argmax

- Protocol : n-1 Compare & Swap

Alice		Bob
$[\![m]\!] \leftarrow [\![a_1]\!]$		
$[\![m]\!] \leftarrow [\![\max(a_1, a_2)]\!]$	C & S $\longleftrightarrow$	$(a_1 < a_2)$
$[\![m]\!] \leftarrow \left[\!\!\left[\max\limits_{j \in [1,i]} a_j\right]\!\!\right]$	C & S $\longleftrightarrow$	$(m < a_i) \Rightarrow \operatorname*{argmax}\limits_{j \in [1,i]} a_j$
$[\![m]\!] \leftarrow \left[\!\!\left[\max\limits_{j \in [1,n]} a_j\right]\!\!\right]$	C & S $\longleftrightarrow$	$(m < a_n) \Rightarrow \operatorname*{argmax}\limits_{j \in [1,n]} a_j$

Argmax

- Protocol : n-1 Compare & Swap

Alice

$[\![m]\!] \leftarrow [\![a_{\pi(1)}]\!]$

$[\![m]\!] \leftarrow [\![\max(a_{\pi(1)}, a_{\pi(2)})]\!]$

$[\![m]\!] \leftarrow \left[\!\!\left[\max_{j\in[1,i]} a_{\pi(j)}\right]\!\!\right]$

$[\![m]\!] \leftarrow \left[\!\!\left[\max_{j\in[1,n]} a_{\pi(j)}\right]\!\!\right]$

$\boxed{\max a_j}$

C & S

C & S

C & S

Bob

$(a_{\pi(1)} < a_{\pi(1)})$

$(m < a_{\pi(i)})$

$\Rightarrow \operatorname*{argmax}_{j\in[1,i]} a_{\pi(j)}$

$(m < a_{\pi(n)})$

$\Rightarrow \operatorname*{argmax}_{j\in[1,n]} a_{\pi(j)}$

$\boxed{\pi(\operatorname{argmax} a_j)}$

Argmax

- Protocol : n-1 Compare & Swap

Argmax

- Protocol : n-1 Compare & Swap sequentially

Argmax

- Protocol : n-1 Compare & Swap

  sequentially

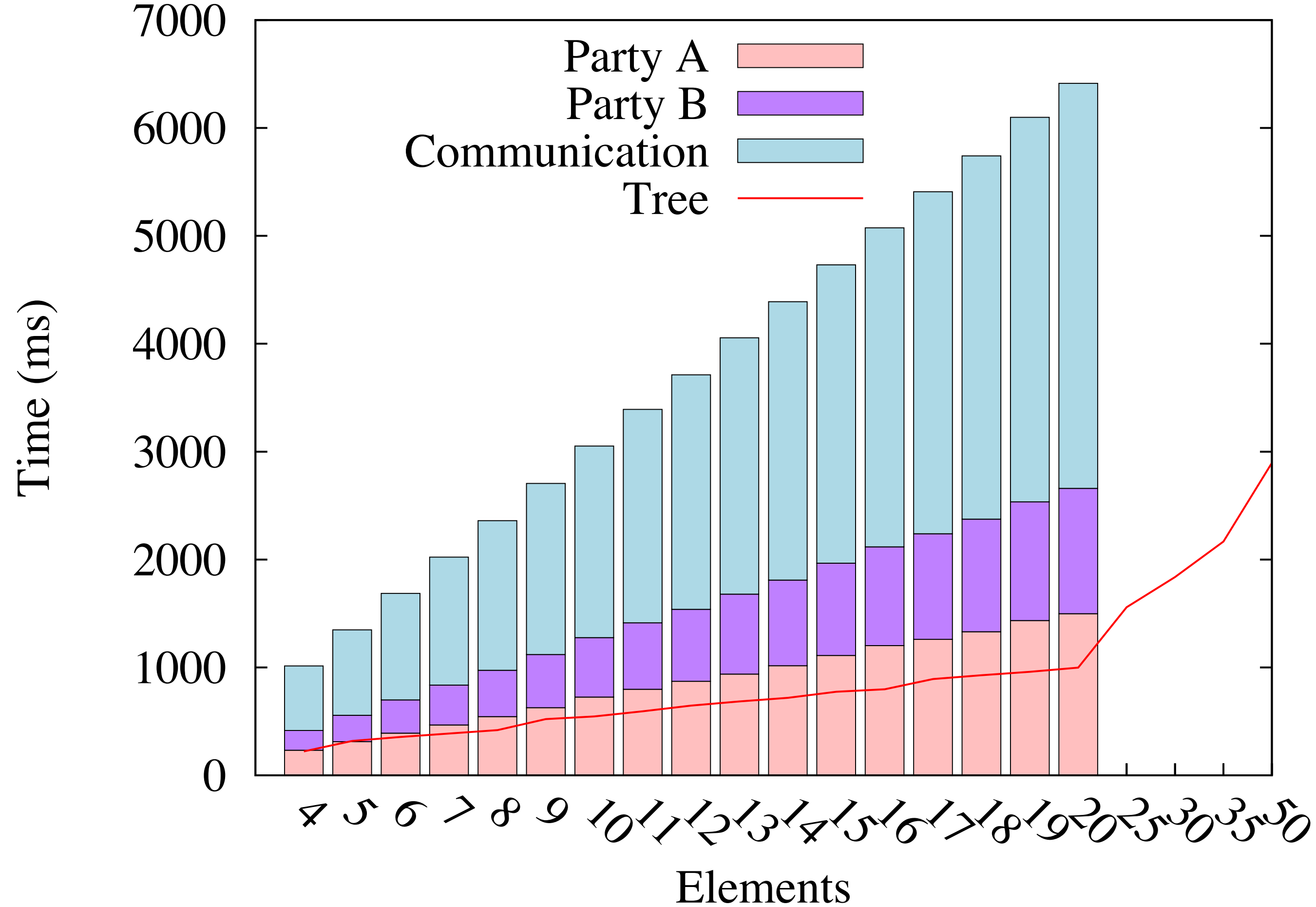
  or in parallel

Argmax

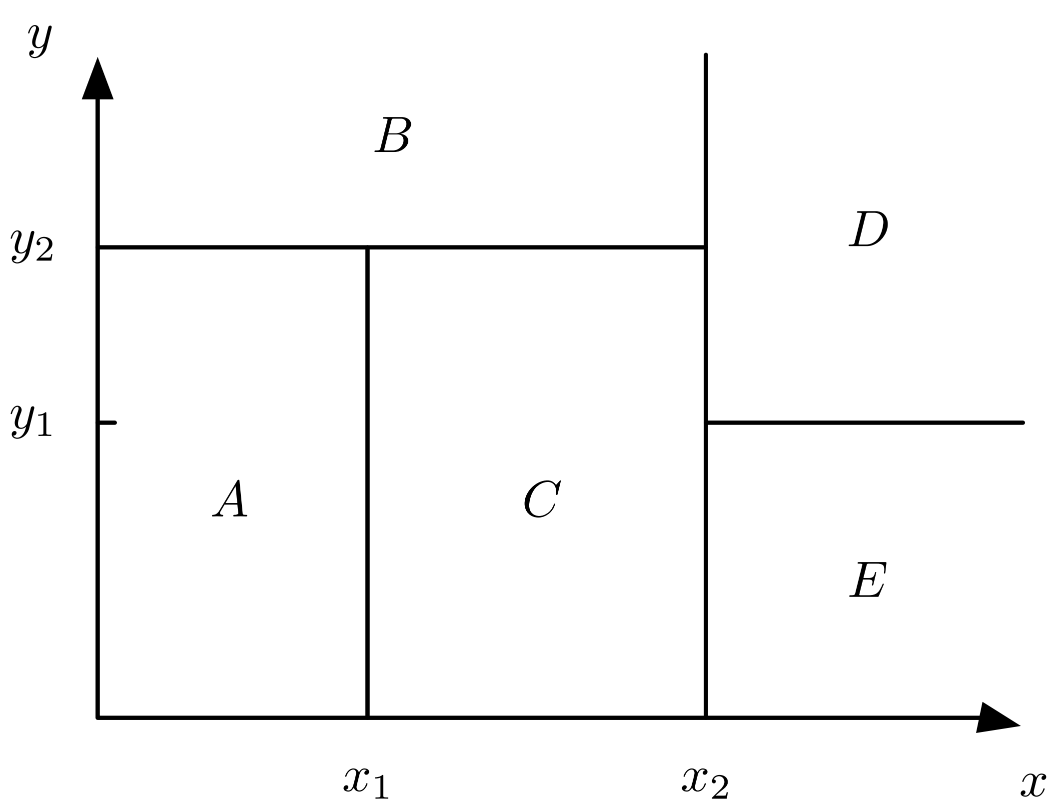
- Protocol : n-1 Compare & Swap

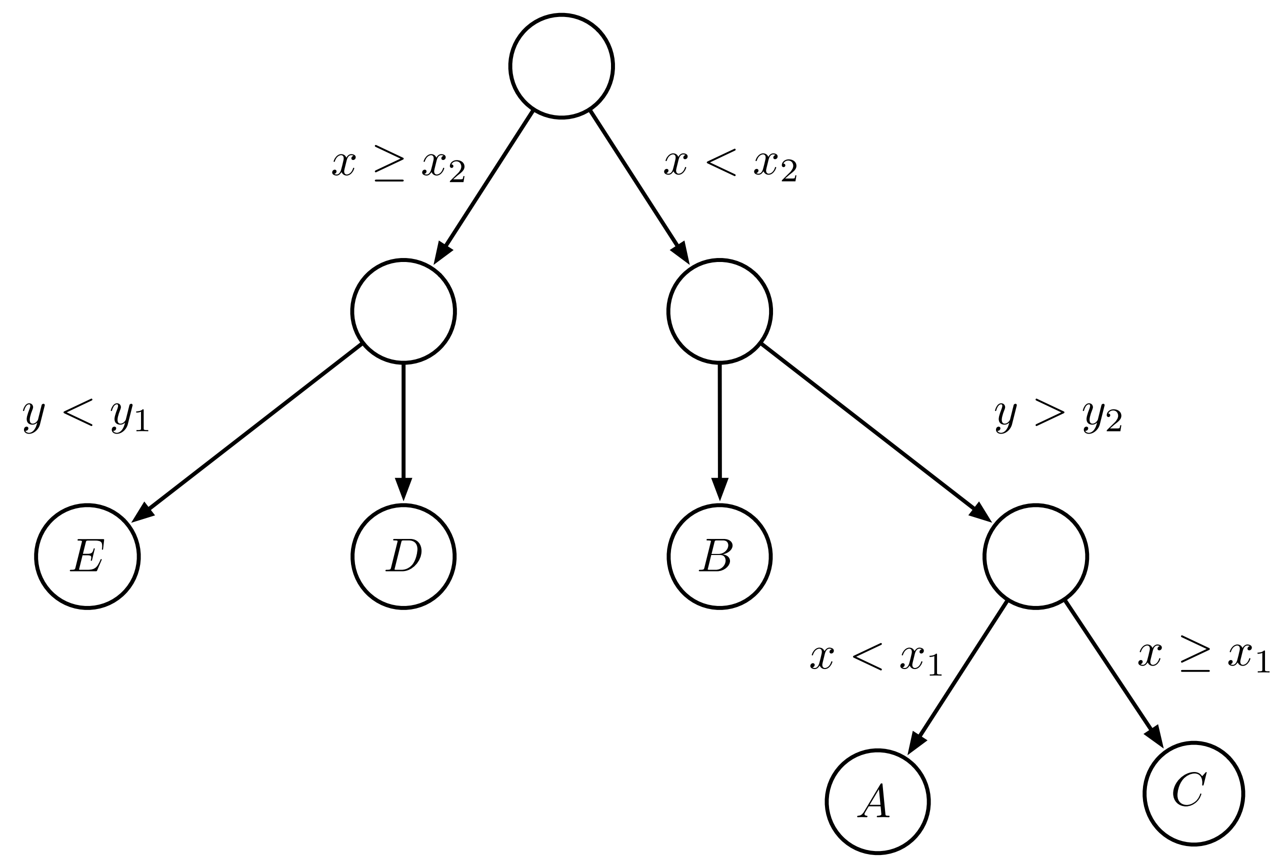
  sequentially

  or in parallel

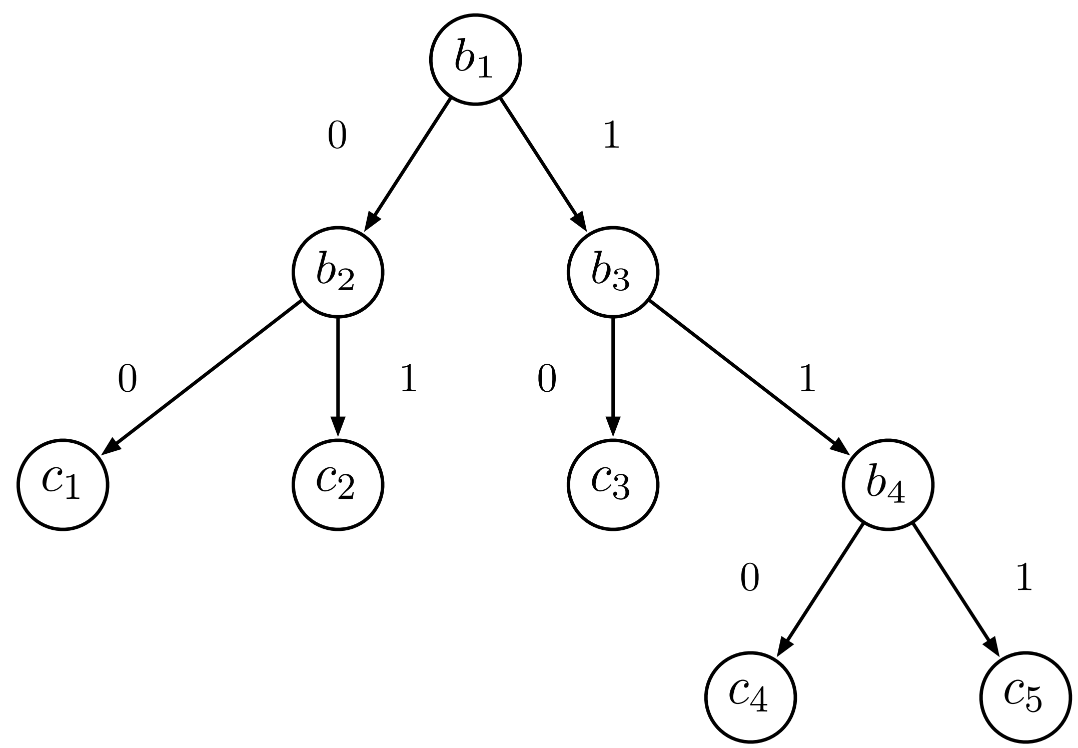


Decision Trees





Decision Trees



$$
\begin{aligned}
P(b_1, b_2, b_3, b_4, c_1, \ldots, c_5) = \quad & b_1 \cdot (b_3 \cdot (b_4 \cdot c_5 + (1 - b_4) \cdot c_4) + (1 - b_3) \cdot c_3) \\
& +(1 - b_1) \cdot (b_2 \cdot c_2 + (1 - b_2) \cdot c_1)
\end{aligned}
$$

Decision Trees

$$P(b_1, b_2, b_3, b_4, c_1, \ldots, c_5) = \quad b_1 \cdot (b_3 \cdot (b_4 \cdot c_5 + (1 - b_4) \cdot c_4) + (1 - b_3) \cdot c_3)$$
$$+ (1 - b_1) \cdot (b_2 \cdot c_2 + (1 - b_2) \cdot c_1)$$

- Polynomial evaluation

  Leveled Homomorphic Encryption

- Binary Variables
- Binary Coefficients ! (SIMD)

Efficient LHE

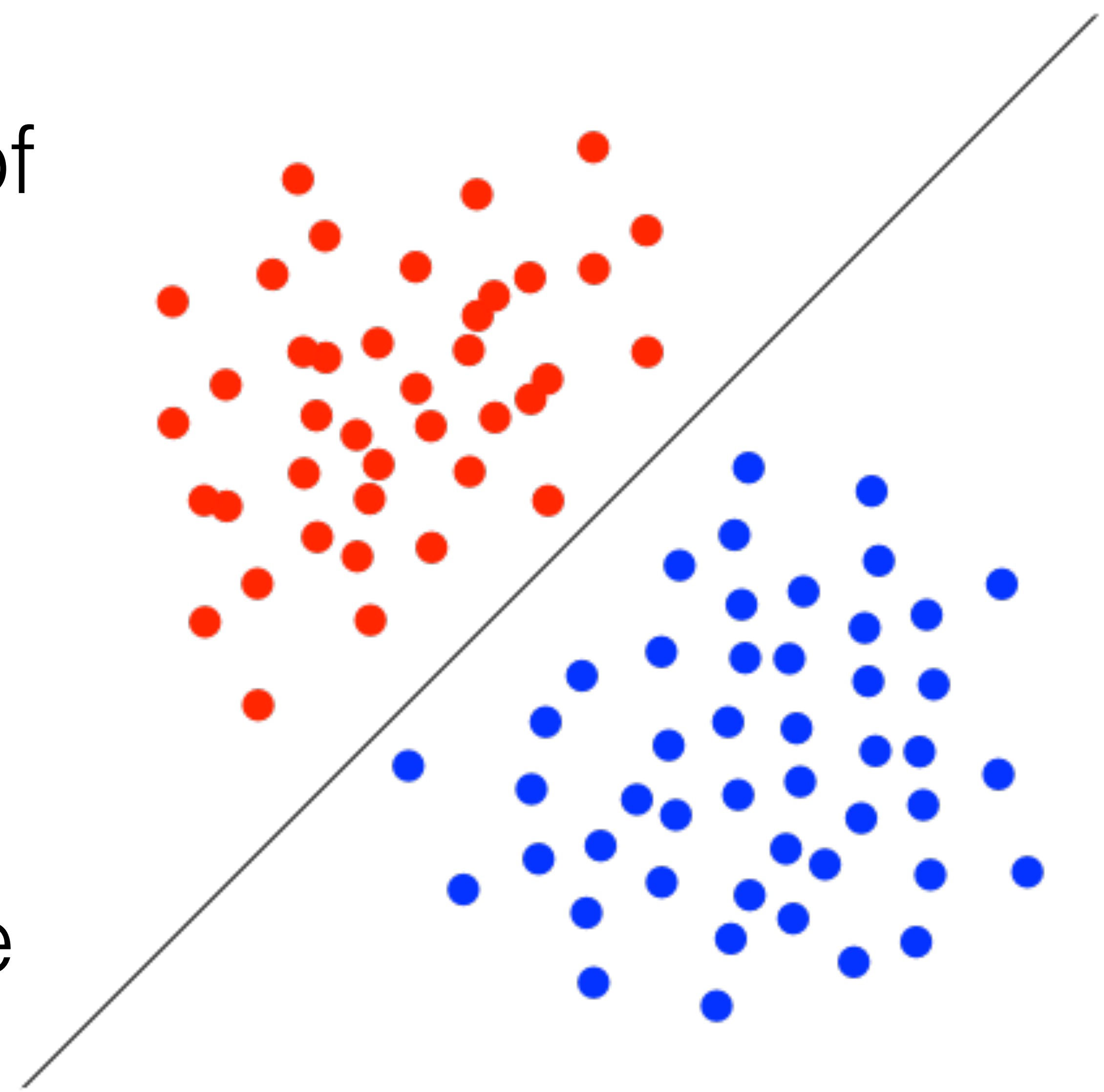
Classifiers

In Practice

- Linear Classifier
- Naïve Bayes Classifier
- Decision Trees

Linear Classifier

- Separate two sets of points
- Very common classifier
- Dot product + Encrypted compare



Linear Classifier

Model Size	Computation		Time / protocol		Total	Comm.	Inter.
	Client	Server	Dot Product	Enc. Comp.			
30	46.4 ms	43.8 ms	194 ms	9.67 ms	204 ms	35.84 kB	7
47	55.5 ms	43.8 ms	194 ms	23.6 ms	217 ms	40.19 kB	7

Evaluation on UC Irvine ML databases
40 ms network latency
2,66 GHz Intel Core i7

Naïve Bayes Classifier

Naïve Bayes Classifier

- Classification $\operatorname*{argmax}_{i \in [k]} p(C = c_i | X = x)$

Naïve Bayes Classifier

- Classification $\underset{i \in [k]}{\operatorname{argmax}} \, p(C = c_i | X = x)$
- Bayes Formula $\underset{i \in [k]}{\operatorname{argmax}} \, \frac{p(C = c_i, X = x)}{p(X = x)}$

Naïve Bayes Classifier

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Naïve Bayes Classifier

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- Bayes Formula $\operatorname*{argmax}_{i \in [k]} p(C = c_i, X = x)$
- Naïve Model $\operatorname*{argmax}_{i \in [k]} p(C = c_i, X_1 = x_1, \ldots, X_d = x_d)$

Naïve Bayes Classifier

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- Bayes Formula $\underset{i \in [k]}{\operatorname{argmax}}\, p(C = c_i, X = x)$
- Naïve Model $\underset{i \in [k]}{\operatorname{argmax}}\, p(C = c_i) \prod_{j=1}^{d} p(X_j = x_j | C = c_i)$

Naïve Bayes Classifier

- Classification $$\operatorname*{argmax}_{i \in [k]} p(C = c_i | X = x)$$
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Naïve Bayes Classifier

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$$\underset{i \in [k]}{\operatorname{argmax}} \quad \log p(C = c_i) \sum_{j=1}^{d} \log p(X_j = x_j | C = c_i)$$

Naïve Bayes Classifier

k	d	Computation Client	Computation Server	Running Time	Comm.	Inter.
2	9	150 ms	104 ms	479 ms	72.47 kB	14
5	9	537 ms	368 ms	1415 ms	150.7 kB	42
24	70	1652 ms	1664 ms	3810 ms	1911 kB	166

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Decision Tree

- Combination of other classifiers
- In this example, linear classifiers
- Linear classifier + ES Switching + Decision Trees

Decision Tree

Tree Specs.		Computation		Time / Protoc.		FHE		Com m.	Inter.
N	D	Client	Server	Lin. Class.	ES Switch	Eval	Decrypt		
4	4	1579 ms	798 ms	446 ms	1639 ms	239 ms	33.51 ms	2639 kB	30
6	4	2297 ms	1723 ms	1410 ms	7406 ms	899 ms	35.1 ms	3555 kB	44

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In conclusion

- Composable building blocks for secure classifiers
- Practical performances

Future work :

- Less roundtrips (work on the protocols)
- More parallelism (work on the implementation)

Questions?